

**ESTIMATING TOTAL MORTALITY FROM LENGTH DATA WHEN SPAWNING  
OCCURS ANNUALLY**

by

John M. Hoenig  
Minnesota Department of Natural Resources  
Box 12, Centennial Office Building  
St Paul, MN 55155, USA

Methods for estimating the total instantaneous mortality rate,  $Z$ , from length-frequency data have been available since Beverton and Holt (1956) derived the formula

$$Z = \frac{K(L_{inf} - L_m)}{L_m - L_c} \quad (1)$$

where  $K$  and  $L_{inf}$  are parameters of the von Bertalanffy growth curve,  $L_m$  is the mean length of fish above the length  $L_c$ , and  $L_c$  is a "knife-edge" recruitment size. This and other approaches, which assume continuous recruitment (i.e. at all times of the year), were reviewed by Pauly (1983) and Hoenig et al (1983).

When recruitment occurs annually (once a year), an analogous discrete model can be derived by equating the mean length with a function of the growth parameters and mortality rate. Thus,

$$\begin{aligned} L_m &= \left( \sum_{i=t_c}^{t_u} N_i L_{m_i} \right) / \left( \sum_{i=t_c}^{t_u} N_i \right) \\ &= \left( \sum_{i=t_c}^{t_u} e^{-Zi} L_{m_i} \right) / \left( \sum_{i=t_c}^{t_u} e^{-Zi} \right) \\ &= \frac{\left( \sum_{i=t_c}^{t_u} e^{-Zi} L_{m_i} \right) (1 - e^{-Z})}{e^{-Zt_c} - e^{-Z(t_u+1)}} \quad (2) \end{aligned}$$

where  $L_{m_i}$  is the mean length at age  $i$ ,  $N_i$  is the number of animals at age  $i$ , and  $t_c$  and  $t_u$  are the youngest and oldest ages fully represented in the sample. Note that  $L_m$ , the sample mean, is the mean of those fish whose ages are fully represented in the sample. This implies that one should choose a left truncation point ( $L_c$ ) that lies in a "valley" between two peaks in a length-frequency distribution. Equation (2) can be solved iteratively using the solution of (1) as a preliminary estimate.

If von Bertalanffy growth equations are substituted for the  $L_{m_i}$  in (2), the formula becomes

$$L_m = L_{inf} - \frac{L_{inf}(1 - e^{-Z}) \left( e^{-t_c(Z+K)} - e^{-(t_u+1)(Z+K)} \right) K t_0}{(1 - e^{-(Z+K)}) \left( e^{-Zt_c} - e^{-Z(t_u+1)} \right)}$$

(Note that an alternative growth model incorporating seasonal oscillations in growth (Pauly and Gaschütz 1979) can also be incorporated into (2). Finally, if there is no reason to believe the older age groups are under-represented, then  $t_u$  can be taken to be infinite and

$$L_m = L_{inf} - \frac{L_{inf}(1 - e^{-Z}) e^{-K(t_c - t_0)}}{1 - e^{-(Z+K)}}$$

Rearranging this to eliminate  $t_0$  gives

$$\frac{L_{inf} - L_c}{L_{inf} - L_m} = \frac{1 - e^{-(Z+K)}}{1 - e^{-Z}} \quad (3)$$

Equation (3) can be solved iteratively using (1) to determine a preliminary estimate.

Using equation (1) as an approximation to (3) results in a positive bias whose severity increases as  $L_c$  approaches  $L_m$  (Table 1).

Table 1. Effect of using continuous spawning-based estimator when reproduction occurs annually.  $K = 0.3 \text{ yr}^{-1}$ ,  $L_{inf} = 40 \text{ cm}$ ;  $t_0 = 0$ . Table gives estimates of  $Z$  derived from equations (1) and (3).

| Lm   | Lc = 10 cm |         | 15 cm   |         | 20 cm   |         |
|------|------------|---------|---------|---------|---------|---------|
|      | eq. (1)    | eq. (3) | eq. (1) | eq. (3) | eq. (1) | eq. (3) |
| 15.0 | 1.5        | .83     | -       | -       | -       | -       |
| 20.0 | .60        | .42     | 1.20    | .71     | -       | -       |
| 25.0 | .30        | .23     | .45     | .33     | .90     | .57     |
| 30.0 | .15        | .12     | .20     | .16     | .30     | .23     |

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