THE GENERAL VON BERTALANFFY GROWTH FUNCTION: POSSIBLE USE IN THE BEVERTON AND HOLT MODEL

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Abstract

Using the concept of generalized Von Bertalanffy growth formula developed by Pauly (1981) and Gaschütz et al (1980) we tried to integrate the basic equation of Beverton and Holt (Gulland, 1983) and to check on changes in the quantitative evaluation of Yield/Recruit. An example is given concerning a tropical fresh water fish population.

Introduction

The yield per recruit, Y/R, is calculated from the following equation which has to be integrated:

$$Y/R = F \int_{t_c}^{t_m} N_{(t)}\ W_{(t)}\ dt \tag{1}$$

in which F is the instantaneous fishing mortality rate
t_c is the age of first capture
t_m is the maximum observed age
$N_{(t)}$ is the number of fishes of age $t > tc$
$W_{(t)}$ is their individual weight

This integral can be algebraically expressed with the following assumptions: The weight-growth is isometric, F is constant between t_c and t_m and M, the instantaneous natural mortality rate is constant between the age of recruitment t_r and t_m (Gulland, 1983). Laurec and Le Guen (1981) have shown that several attempts have been made to avoid these conditions of use of the Beverton and Holt model, including the isometry.

The recent generalization of the Von Bertalanffy growth function as developed by Gaschütz et al involves the possibility of allometry, and also the occurrence of seasonal variations in the growth rate which have to be considered in the evaluation of Y/R.

This paper is involved with trials in this matter including the possibility of description of annual shrinkage in weight which can be noticed with appropriate studies of weight growth pattern.

Calculations

In equation (1) $N_{(t)}$ has to be expressed by the usual way:

$$N_{(t)} = R\ e^{-M(t_c - t_r)}\ e^{-(F+M)(t-tc)} \tag{2}$$

In the basic equation of Beverton and Holt it is assumed that

$$W_{(t)} = Winf[1 - e^{-K(t-t_o)}]^3 \quad \text{or} \quad W_{(t)} = Winf \sum_0^3 Un\ e^{-nK(t-t_o)} \tag{3}$$

with n = 0, 1, 2, 3 and Un = 1, -3, 3, -1.

The seasonal variations of growth rate are described when writing the equation of Von Bertalanffy as follows (Gaschutz et al 1980):

$$L_{(t)} = Linf[1 - e^{-(KD(t-t_o) + KD(C/2PI)SIN\ 2PI(t-t_s))}]^{1/D} \quad (4)$$

Assuming that the growth is allometric, the weight growth may be described by

$$W(t) = Winf[1-e^{-((3/b)\ KD(t-t_o) + KD(C/2PI)SIN\ 2PI(t-t_s))}]^{b/D} \quad (5)$$

in which D is a parameter depending on physiological features of the fish stock (Pauly, 1981), C is generally between 0 and 1 according to the importance of seasonal changes in growth rate and t_s is the first age for which the seasonal variation of growth rate occurs in the life of the fish.

The assumption that C = 1 means that growth in length ceases completely once every year. Usually, during a stop in growth in length a shrinkage in weight can be noticed, even in tropical fishes. A shrinkage of about 10-15% has been observed in some tilapia populations (Moreau, 1979).

Several computation trials have been performed in order to describe this seasonal shrinkage by a correct value of C. It appears that C has to be related to the age t such that

$C_{(t)} = Co + f_{(t)}$. This can be written as

$$C_{(t)} = Co + a(W_{(t)} / Winf) \quad (6)$$

in which a and Co are constants depending on the fish population; $W_{(t)}$ is the weight at age t calculated as follows

$$W_{(t)} = Winf\ [1 - e^{-K(t-t_o)}]^b$$

The value of $C_{(t)}$ has to be incorporated into equation (5) instead of the assumed constant value of C.

We can refer to the equation (5) as a "general Von Bertalanffy weight growth function": GVBWGF. Without seasonal changes, this equation (5) is simplified to the equation (3).

The equations (2) and (5) can be used to rewrite equation (1) and after rearranging, we obtain Y/R as follows:

$$Y/R = F\ Winf\ e^{-M(t_c-t_r)} \int_{t_c}^{t_m} e^{-(F+M)(t-t_c)} [1-e^{G_{(t)}}]^{b/D}$$

in which

$$G_{(t)} = -(3/b)\ KD(t-t_o) + (CKD/2PI)\ SIN\ 2PI\ (t-t_s)$$

and C increases with age according to equation (6).

The numerical value of the integral can be calculated by several methods which can be programmed in BASIC on any microcomputer (Moreau et al 1984); a listing is available from the author.

Example

Figure 1 shows the observed linear growth in a population using the usual Von Bertalanffy function for which D = 1 without seasonal variation. In addition, for this population of <u>Tilapia rendalli</u> in a tropical high altitude lake, it was possible to consider seasonal variations of the growth (D remained 1).

Figure 2 shows the seasonal changes in growth in weight. The shrinkages are included because the points refer to observed data on weight growth for the fish stock. Iterative trials showed that the best simulation (it has to be noticed that we do not say "fitting") of the weight growth is obtained with the following value of C:

$$C = C_{(t)} = 2 + (2W_{(t)}/Winf)$$

In relation to these results we give the curves showing the variations of Y/R according to t_c, at several levels of fishing effort. Figure 3 shows Y/R curves for Tilapia rendalli calculated in the usual way using the following parameters: Winf = 965 g, K=0.256, t_o = 0.08, b=3, t_r = 1 year and t_m = 6 years and M = 0.50 (as previously estimated (Moreau, 1979).

Figure 4 shows Y/R curves for Tilapia rendalli which allow for allometric growth and an oscillating growth curve using the following parameters : Winf = 1215 g, K=0.286, t_o=-0.091, b=2.98, D=1, t_s=0.314 and C as derived from Eq.(3) and with no changes for the other parameters.

When compared, figs. 3 and 4 show clear changes in the shape of the curves and in the values of Y/R when seasonal variations in growth are included (fig. 4). The influence of weight shrinkages is the most important during the first two years after t_c and increases with the level of fishing effort i.e. with increasing values of F.

Figure 4 shows clearly that for this particular Tilapia rendalli stock the optimal value of t_c should be 2.5 years irrespective of what the fishing effort is. A little change (increase a decrease) of t_c should considerably modify the expected value of Y/R and we can reasonably expect a poor situation for the fisheries for this stock, keeping in mind that the mean age of first capture has actually been about 2 years since 1977 (Moreau, 1979).

Conclusion

The availability of micro-computers enables fisheries scientists to include in their predicting models more information than previously and to more closely approach the biological realities. This is the main usefulness of the present trials. But we have to keep in mind that the accuracy of the predictions of Y/R depend upon the choice of equation used (including that given here) and also, let us say mainly, on the accuracy of the evaluation of the entered parameters. Without research to give optimal quality to the evaluation of each parameter, it is pointless to improve sophisticated expressions for Y/R and ad hoc computing programs. Particular care is needed in the evaluation of the natural mortality rate, M, in short-lived species usually found in tropical waters (Moreau, 1979).

References

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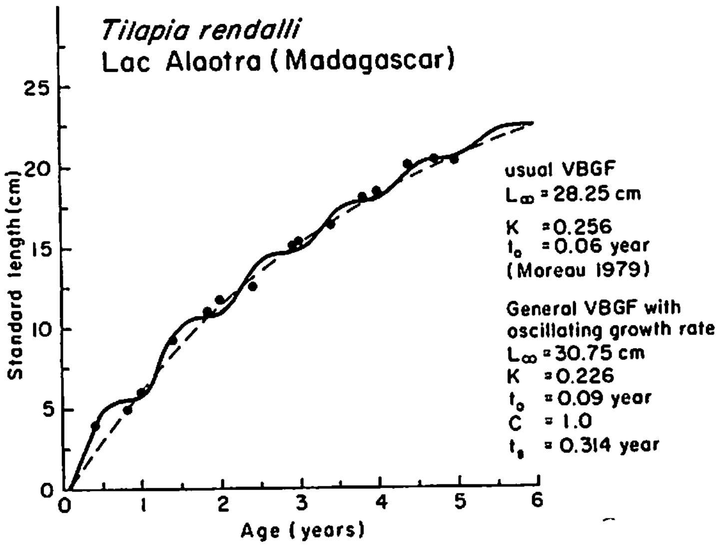

Fig. 1. Length growth of *Tilapia rendalli* in the Alaotra Lake (Madagascar). The dashed line is described by the usual V.B.G.F. when Linf = 28.25 cm, K = 0.256 and t_0 = 0.08. The continuous line represents the seasonally oscillating V.B.G.F. when Linf = 30.25 cm, K = 0.286, t_0 = 0.09, t_s = 0.314 and C = 1.

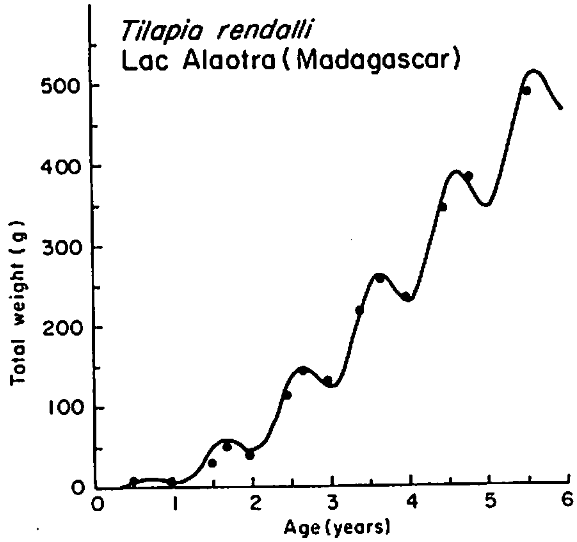

Fig. 2. Weight growth of *Tilapia rendalli* in the Alaotra Lake. The points refer to observed date on growth in weight including the shrinkage during each linear growth stop. These shrinkages are correctly simulated when assuming C = C_t = 2 + ($2W_t$/Winf) and Winf = 0.0448 • Linf^3 = 1,215 g, K = 0.26, t_0 = 0.09 and t_s = 0.314.

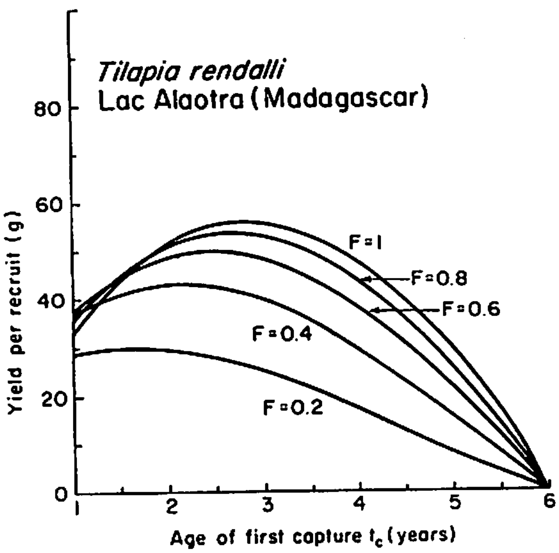

Fig. 3. Yield per recruit computations using the usual V.B.G.F. for different levels of fishing activity (0.2 < F < 1).

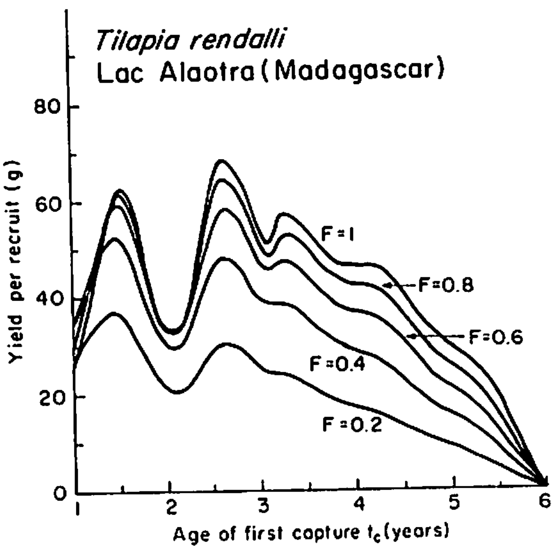

Fig. 4. Yield per recruit computations using the V.B.G.F. including the seasonal pattern of length growth and yearly weight shrinkages.