STATISTICAL METHOD OF ESTIMATING THE SIZE AT FIRST MATURITY IN FISHES

By

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The reproductive process in all animals commences once they attain maturity at a particular size, which depends upon environmental and ecological factors and the biological features of the animal. The size at maturity is subject to variation between species and within species. Thus all fishes of the same cohort or size need not attain maturity at some fixed age or length. At present, methods are available to estimate the exact average size of fish at first maturity by graphing the percentage cumulative frequency curve of mature fishes against size, or by plotting the mean condition factor (K_n) against size. These methods give a single estimate of the mean size at first maturity in the population. Statistically this estimate may be without any confidence limits. A method of obtaining such limits is explained in the present paper.

Methodology

Finney (1971) has detailed several statistical methods for estimating lethal dose (LD_{50}) for animals under quantal responses subject to certain constraints. For fishes, using the cumulative percentage frequency curve of mature fish in the sample, the size at first maturity is the size corresponding to 50% maturity. This principle is embodied in the Spearman-Karber method of estimating mean LD_{50} under quantal responses with confidence limits for estimating the mean size at first maturity in the fish population. To apply this method, the sample should include all possible sizes of fish including both immature and mature specimens. The different maturity stages of sampled fish are recorded under different length groups with equal class-width. Then, using the formula of Spearman-Karber, the log size at first maturity (m) of a species is given by

$$m = x_k + \frac{X}{2} - \left(X \sum p_i\right) \quad (1)$$

where x_k = last log size at which 100% of fish are fully mature

X = log size increment = $x_{i+1} - x_i, i = 1, 2, \ldots k-1$

and x_o = last log size at which no fishes are fully mature

r_i = number of fully mature fish in the i^{th} size group

p_i = proportion of fully mature fish in the i^{th} size group

$p_i = r_i/n_i$, if $n_i \neq n_{i+1}$ for $i = 1, 2, \ldots k-1$

and $p_i = r_i/n$, if $n = n_i = n_{i+1}$ for $i = 1, 2 \ldots k-1$

Thus, the mean size at first maturity is given by

$$\text{antilog}(m) = M \quad (2)$$

The $(1 - \alpha)$ % confidence limits in log sizes are given by

$$m \pm Z\frac{\alpha}{2}\sqrt{Var(m)} \quad (3)$$

where $Z\frac{\alpha}{2}$ = confidence coefficient at α level of risk,

and $\sqrt{Var(m)}$ = SE(m) = standard error of estimate of mean log size

But, $Var(m) = X^2 \Sigma_i \left[\frac{p_i \cdot q_i}{n_i - 1}\right]$, if $n_i \neq n_{i+1}$, i = 1,2 .....k-1

and $q_i = 1 - p_i$

and $Var(m) = \frac{X^2}{n^2(n-1)} \Sigma_i \left[r_i(n-r_i)\right]$, if $n_i = n_{i+1} = n$, for i=1,2 ....k-1

if α = 0.05, then 95% confidence limits are given by

$$\text{antilog}\left[m \pm 1.96\sqrt{X^2 \Sigma_i \left\{\frac{p_i \cdot q_i}{n_i - 1}\right\}}\right] \quad (4)$$

if $n_i \neq n_{i+1}$, for i = 1,2 ....k-1

$$\text{or, antilog}\left[m \pm 1.96\sqrt{\frac{X^2}{n^2 (n-1)}\Sigma\left\{r_i(n-r_i)\right\}}\right] \quad (5)$$

if $n_i = n_{i+1} = n$, constant for i = 1,2, ...k-1

Example.

Table 1 gives the length frequency data for male R. kanagurta up to maturity stage III, which is the final maturation stage, at which testes are opaque, white and flat and extend from 1/3 to 2/3 of the length of the body cavity (Azad, 1983). Since the lengths are not normally distributed, the mid-lengths are converted into log values to fulfil one of the conditions to use equation (1). The mean size (M) at maturity is estimated as the size where 50% of the fish are mature. In Table 1 the total number of fish which have passed stage III (r_i) are given, from which proportion of fully mature fish (p_i) are calculated, starting from the 17-19 cm length group. This is to fulfil another condition for applying equation (1), to ensure 0 and 1 as the proportions of fully mature fish corresponding to log sizes x_o and x_k respectively and thus the data are truncated from the original data. During the course of the study one may or may not select equal numbers of fish for each size and accordingly the third term of equation (1) changes. Table 1 gives the various steps in estimating the size at first maturity using equation (1).

This gives, $m = 1.415 + \frac{0.04}{2} - [0.04\ (3.02)]$

or, m = 1.3142

From (2), M = antilog (1.3142)

= 20.62 cm

From equation (4), the 95% confidence limits are given by

antilog $[1.3142 \pm 1.96\sqrt{(0.04)^2\ (0.0211)}]$

or antilog $[1.3142 \pm 0.0114]$

The logarithmic confidence limits are therefore m_L = 1.3028 and m_u = 1.3256, and the limits are

M_L = antilog (1.3028) = 20.08 cm

and M_u = antilog (1.3256)= 21.16 cm

Results and Discussion

Equation (2) gives the mean size at first maturity as 20.62 cm for the sample of male R. kanagurta. According to Azad (1983) based on a cumulative percentage frequency curve, males attain maturity at size 19.8 cm and using K_n

Table 1: Length-frequency distribution and maturity stages of male Rastrelliger kanagurta and computations for estimation of mean size at first maturity with confidence limits.

Length group (cm)	Mid length (cm)	Log mid length (x_i)	# of fish sampled (n_i)	Number of fish at maturity Stage I	Stage II	Stage III	total # of fully mature fish (r_i)	proportion of fully mature fish (p_i)	$x_{i+1}-x_i=X$	$q_i=1-p_i$	$\frac{p_i \cdot q_i}{n_i-1}$
13–15			5	5	0	0					
15–17			12	10	2	0					
17–19	18	1.2553	15	7	6	2	0	0	0.0458	1.0	0
19–21	20	1.3010	25	5	7	4	9	0.36	0.0414	0.64	0.0096
21–23	22	1.3424	43	3	0	6	34	0.79	0.0378	0.21	0.0040
23–25	24	1.3802	16	0	0	2	14	0.87	0.0348	0.13	0.0075
25–27	26	1.4150	6	0	0	0	6	1.00		0	0
Total								3.02			0.0211

values he opines that the majority of male fish mature as they attain a length of 20 cm. Rao and Ramamohana (1967) have indicated that R. kanagurta caught by shore-seine off Mangalore mature at 21.7 cm and Rao et al. (1962) have estimated the length at maturity at 20 cm. These discussions do not give any definite idea of mean size at first maturity of R. kanagurta in the absence of statistical limits. Reports are available where the Indian Mackerel matured at 19.0 cm off Calicut (Devanesan and John, 1940), at 22.4 cm off Karwar (Pradhan, 1956) and at 21–22 cm off Karwar (Radhakrishnan 1962). Using scales and otoliths, Sheshappa (1958, 1969) estimates that the Indian mackerel attain maturity at 20 cm but based on scales alone he concluded that l_m is at 22 cm. The confidence limits of 20.1 cm to 21.2 cm, with a mean population maturity size of 20.6cm, as estimated here, give a better idea of size at first maturity using length frequency data than statements based only on point estimates.

The chief attraction of this method is that it does not require the percentage distribution of fish over different maturity stages in various length groups and no graphic representation is required to estimate the maturity size. The interpretation of mean size at first maturity says that when the males of R. kanagurta attain an average size of 20.6 cm, most of the fish are at the middle of stage III and that full maturity is attained after completion of maturity stage III.

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