Estimation of Yield per Recruit When Growth and Fishing Mortality Oscillate Seasonally

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Abstract

A general form of the yield equation is presented. This is used to investigate the effects of seasonal growth oscillations on estimation of yield per recruit. The results show that these effects will be strongest in short-lived organisms such as penaeid shrimps.

Introduction

Beverton and Holt's (1957) yield-per-recruit (Y/R) formula is based on the assumption that body weight grows according to the ordinary von Bertalanffy Growth Function (VBGF), that the gear selection is of the knife-edge type and that fish mortality is constant throughout the year.

The objective of this note is to demonstrate how to calculate yield per recruit in the general case and to apply this to the special case of the seasonalized VBGF and a seasonally variable fishing mortality. [Thus, knife-edge selection will be assumed; for a detailed discussion on its effects on Y/R computation, see Silvestre et al. (1991)].

The type of derivation given below has been given earlier by many authors and they are by no means presented as original ideas. Thus, Thompson and Bell (1934) developed a very general yield-per-recruit model and this contribution may be seen as a special case of their model.

The General Case

Let C[t] denote the numbers from one cohort of fish caught during a short time period, (dt), and let w[t] be the average body weight during that time period. Let the life span of the species in question be of length n*dt. For example, if dt is one week and the life span is two years, then n = 104. The yield from a cohort during its life is then

$$Y[t] = \sum_{t=1}^{104} C[t] * w[t] \qquad \text{...1)}$$

This is the simple and general formula for yield.

If equation (1) is divided by the number of recruits on both sides, yield per recruit is the result.

The Beverton and Holt yield-per-recruit model of 1957 is the special case where w[t] is given by the ordinary von Bertalanffy equation and C[t] is given by the usual catch equation with knife-edge gear selection.

Thus, if the ordinary VBGF is replaced by its seasonalized version, the result is a seasonalized version of Beverton and Holt's yield-per-recruit model.

Indeed, any assumption on the fishing mortalities can be used to calculate C[t] and this again leads to the general model of Thompson and Bell (1934).

Equation (1) represents an approximation to the exact mathematical expression, which ideally should be calculated using integral calculus, as was done in the case of the Beverton and Holt formula. However, by reducing the duration of the time step (dt), any precision can be achieved when estimating (1).

Thus, from the theoretical point of view, it is easy to handle the yield (-per-recruit) calculations for any assumption on growth or fishing mortality. From the practical point of view, it is complicated only when one lacks access to a computer and appropriate software.

Thus, the actual calculation of one point on the yield curve in the example given above would require 104 calculations of C[t] and w[t] plus the appropriate sum of products.

To obtain a reasonable description of a yield (-per-recruit) curve, we would need some 10 to 20 points, depending on the shape of the curve. Thus, the computational work involved can be considerable. To reduce this, one could increase the time step to say, dt = one month or one year. This will be appropriate only for long-lived species, however.

The Special Case

Let M[t] and F[t] denote natural and fishing mortality, respectively, and let Z[t] = F[t] + M[t] denote total mortality.

The special case considered here deals with the case where

1. w[t] is assumed to follow the seasonalized version of the VBGF; and
2. no assumptions are made as to M or F.

Thus,

$$C[t] = N[t] * F[t] * (1- \exp(-Z[t]*dt)) / Z[t] \quad ...2)$$

where N[t] is the number of survivors form the cohort at the beginning of the time period [t, t+dt].

N is calculated using the exponential decay model

$$N[t] = N[t-1] * \exp(-Z[t-1] * dt) \quad ...3)$$

Body weight is then calculated using

$$w[t] = a * L[t]\hat{}b \quad ...4)$$

where

a and b are the usual length/weight parameters and

$$L[t]=L_\infty*(1-\exp[-K(t-t_0)-CK/2\pi)\sin\{2\pi(t-t_s)\}]) \quad ...5)$$

where L_∞, K and t_0 are the von Bertalanffy growth parameters, C is the amplitude of the seasonal oscillation in growth rate and t_s is the so-called "summer point" (Pauly and Gaschütz 1979)[a].

Definition of T-zero

The definition of t_0 is problematic and important (see, e.g., Somers 1988).[a] Obviously it is of major interest to compare the Y/R curve based on seasonalized growth with that based on the ordinary VBGF. In the examples given below, it is shown that the Y/R curves are very sensitive to the value of t_0 for short-lived, fast-growing species with pronounced seasonality of growth.

For the ordinary von Bertalanffy growth curve, t_0 is defined by $L[t_0] = 0$, i.e., the hypothetical (negative) age at which the fish has a length of zero. In the seasonalized growth model, we could choose t_0 so that length at age t_0 is the same for the ordinary and the seasonalized growth model. Thus, definition 1 : $t_{02} = (C/2*\pi) * \sin(2*\pi*(-t_s)) + t_{01}$ where t_{01} is the t_0 from the ordinary VBGF model.

An alternative definition is to use the same t_0 for both curves; this leads to definition 2 : $t_{02} = t_{01}$. The latter definition has certain advantages, demonstrated in the example below.

Seasonal Variation in Fishing Mortalities

Fishing mortalities F[1], F[2],..., F[t], ... are calculated as the product of a multiplication factor, X, and the reference fishing mortality

$$F[t] = X * Fr[t]$$

so that, for example, X = 0.8 indicates a 20% reduction of the reference fishing mortality and X = 2 indicates a doubling of the reference fishing mortality; X is the abscissa for the yield or yield/recruit graph. Seasonality is introduced by assigning seasonal values to the reference F.

To illustrate this, we consider an example where dt is a quarter of the year and the life span of the species is four years. Then the array of fishing mortalities becomes a set of 16 elements, F[1], F[2], ..., F[16]. To emphasize the seasonality aspect they are arranged in four age groups:

Age group	Jan-Mar	Apr-Jun	Jul-Sep	Oct-Dec
0	Fr[1]	Fr[2]	Fr[3]	Fr[4]
1	Fr[5]	Fr[6]	Fr[7]	Fr[8]
2	Fr[9]	Fr[10]	Fr[11]	Fr[12]
3	Fr[13]	Fr[14]	Fr[15]	Fr[16]

Assigning the numerical values:

Age group	Jan-Mar	Apr-Jun	Jul-Sep	Oct-Dec
0	1	0	0.5	1
1	1	0	0.5	1
2	1	0	0.5	1
3	1	0	0.5	1

would mean that all age groups are subject to the same fishing pressure and that fishing is at its maximum from October to March, takes on half of the maximum from July to September, while fishing is closed from April to June. The rows need not be identical. Assigning, for example, the values:

Age group	Jan-Mar	Apr-Jun	Jul-Sep	Oct-Dec
0	0.5	0	0.25	0.5
1	1.0	0	0.50	1.0
2	1.0	0	0.50	1.0
3	1.0	0	0.50	1.0

will give the same interpretation as above, except that the O-group has only half of the fishing pressures as the other age groups (e.g., because of gear selection).

[a]Editor's note: Somers (1988) showed that equation (5) generates biased estimates of t_0 when fitted to length-at-age data. However, the corrected version of (5), provided by Somers, lacks an explicit solution for t_0 (see Soriano and Jarre 1988); the FiSAT software (see p. 47), which will incorporate a generalized Y/R model, will take account of this problem.

Numerical Example

To demonstrate the actual calculations, we consider a hypothetical example with the parameters shown in Table 1. This is a fast-growing species with a short life span (two years); the following constants are used: time step (dt) = 0.01923 (year; i.e., one week); L_∞ = 100; growth parameter K = 1.5 year^{-1}; natural mortality (M) = 3.0 year^{-1}; amplitude of seasonal growth oscillations (C) = 1.0; summer point (t_s) = 0.8 year; t_0 (definition 1) = 0.151 year; condition factor (q) = 0.01; exponent of L/W relationship (b) = 3.0.

Table 1. Multiplicators of F used for the first hypothetical example.

Age group	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
0	1.0	1.0	1.0	1.0	1.0	0.5	0.5	0.5	0.0	0.0	1.0	1.0
1	1.0	1.0	1.0	1.0	1.0	0.5	0.5	0.5	0.0	0.0	1.0	1.0

With a time step of one week, all quantities must be calculated 104 times for each point on the yield-per-recruit curve. In this example, we have ignored the gear selection/availability curve and assumed all ages to have the same probability of capture; selectivity is ignored here only to allow us to concentrate on the subject of this note.

We start by calculating the body lengths and body weights using equations (5) and (4), respectively. The results are indicated in Table 2.

Table 3 documents the calculation of yield using the multiplication factor X = 0.9, using equations (1), (2) and (3). Note that the calculations are expressed "per 1,000 recruits".

The total of the products C*W is the total yield. Thus, the point (X; Y/R) = (0.9; 638,155) is one point on the yield curve. To fit the curve we need more than one point, so all the calculations of Table 3 have to be repeated for a suitable range of X-values.

Table 4 shows 20 Y/R-values for different X-values; with these, we are able to draw the Y/R curve (lower curve in Fig. 1A).

Table 2. Length and weight at age by week values required for Y/R calculations.

Week no.	Age (year)	Length	Weight
1	0.0096	1.83	0.06
2	0.0288	5.15	1.36
3	0.0481	8.04	5.19
4	0.0673	10.53	11.67
5	0.0865	12.65	20.23
14	0.2596	19.93	79.20
15	0.2788	19.98	79.73
16	0.2981	19.98	79.82
17	0.3173	19.99	79.87
101	1.9327	94.21	8,361.20
102	1.9519	94.47	8,432.05
103	1.9712	94.71	8,495.99
104	1.9904	94.92	8,553.27

Discussion

The results presented above are straightforward enough. However, when seasonal oscillations are considered, the conversion of length into weight is more complicated than equation (4) indicates. Fish may lose weight, especially in conjunction with spawning, but negative increases will not occur in the case of length growth. For juvenile fish we expect a better match between growth in weight and length than for adult fish.

Seasonality of growth is important for the yield calculations only in the case of short-lived species with one or at most two spawning periods during their life (for example, penaeid shrimps).

To illustrate this, two hypothetical examples are presented: species A, which is a fast-growing species with a short life span (the same species as that of Table 1) and species B, which is a slow-growing species with a long life span.

Thus, for species A, seasonality is more important than for species B. The parameters for the two examples are given in Table 5.

The growth parameters for the ordinary VBGF and of the seasonalized VBGF only deviate with respect to their values of t_0, which were chosen so that both growth curves started with the same length at the same time of the year.

Fig. 1 compares the seasonalized and the nonseasonalized growth curves. Notice that with definition 1 for t_0, the seasonalized growth curve will always be below the nonseasonalized curve. Therefore, using definition 1, the seasonalized Y/R curve always remains below the nonseasonalized one.

Fig. 2A shows the two Y/R curves; as expected the seasonalized curve is far below its nonseasonalized counterpart. The difference of the two curves is considerable. However, what is seen in Fig. 1A probably represents an extreme case, because growth is very fast, mortality very high, and the amplitude of the growth oscillation is set at its maximum value (C=1).

For a slow-growing species (Example B), the difference is rather insignificant as appears from Fig. 3A.

Changing the definition of t_0 thus has a crucial impact on the curves. Suppose we had observed weight at age corresponding to the seasonalized curve in Fig. 1B and

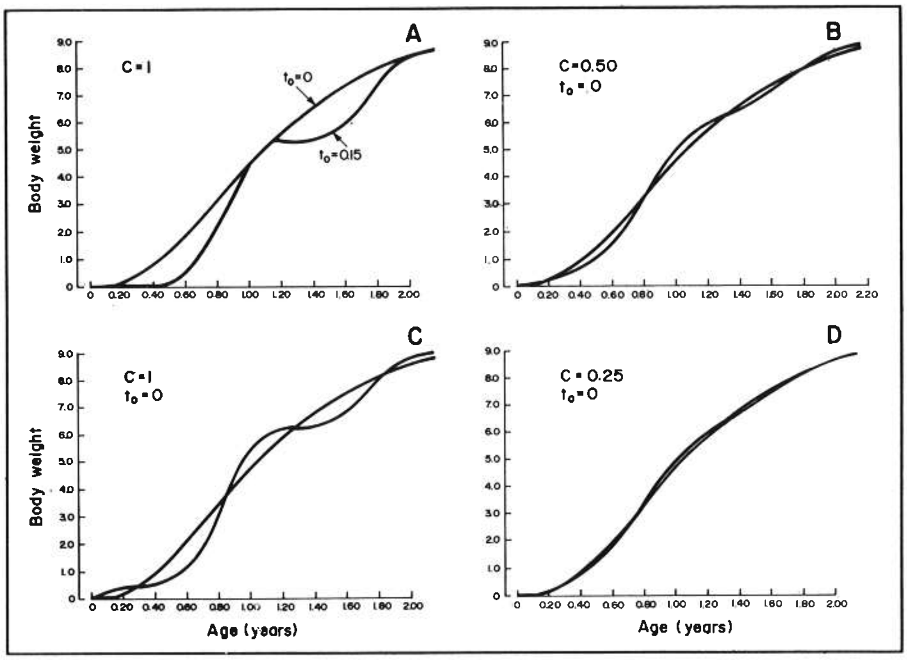

Fig. 1. Comparison of seasonalized and nonseasonalized growth curves for a short lived fast growing species.
Example A of Table 5.
A: C = 1.00, definition 1 of t_o (t_o = 0.151)
B: C = 1.00, definition 2 of t_o (t_o = 0.0)
C: C = 0.50, definition 2 of t_o (t_o = 0.0)
D: C = 0.25, definition 2 of t_o (t_o = 0.0)

Table 3. Example of a calculation of Y/R with the multiplication factor X = 0.9 (F_{ref} = 1.0; F = 0.9; Z = 3.9).

			N(1-exp(z))		
Week	exp(-Z)	N	Z	C	C*w
1	0.928	10,000.00	185.27	166.75	10.2
2	0.928	9,277.43	171.89	154.70	211.0
3	0.928	8,607.08	159.47	143.52	745.0
4	0.928	7,985.16	147.94	133.15	1,553.4
5	0.928	7,408.18	137.25	123.53	2,499.6
14	0.928	3,771.92	69.88	62.90	4,981.1
15	0.928	3,499.38	64.83	58.35	4,652.1
16	0.928	3,246.52	60.15	54.13	4,320.8
101	0.928	9.46	0.18	0.16	1,318.7
102	0.928	8.77	0.16	0.15	1,233.7
103	0.928	8.24	0.15	0.14	1,153.3
104	0.928	7.55	0.14	0.13	1,077.2
TOTAL	638,155.1				

Table 4. Final result of Y/R calculations. Each Y/R value requires a calculation such as illustrated in Table 3.

X	Y/R
0.00	0.0
0.10	128,220.3
0.20	237,455.5
0.30	330,091.0
0.40	408,218.4
0.50	473,672.5
0.60	528,063.5
0.70	572,805.2
0.80	609,138.9
0.90	638,155.1[a]
1.00	660,812.1
1.10	677,951.5
1.20	690,313.1
1.30	698,547.0
1.40	703,224.3
1.50	704,846.5
1.60	703,854.2
1.70	700,634.0
1.80	695,525.4
1.90	688,825.5

[a]See Table 3.

used these to estimate the parameters of the ordinary VBGF. Then we would have obtained the curve shown in Fig. 1B and not that of Fig. 1A.

This is a good reason for using definition 2 of t_o rather than definition 1. Fig. 2B shows the Y/R curves when definition 2 of t_o is used; the nonseasonalized curve is now the lower curve.

Bearing in mind the difficulties in estimating t_o (one must have absolute/length or weight-at-age data to estimate this parameter), this shows that one should be very cautious when interpreting Y/R curves for fast-growing species with high mortality.

To illustrate the sensitivity of results to the value of C, two more simulations were made, using the C-values 0.5 and 0.25 (and using definition 2 for t_o). The growth curves are shown in Figs. 1C and 1D and the corresponding Y/R curves are given in Figs. 2C and 1D, respectively.

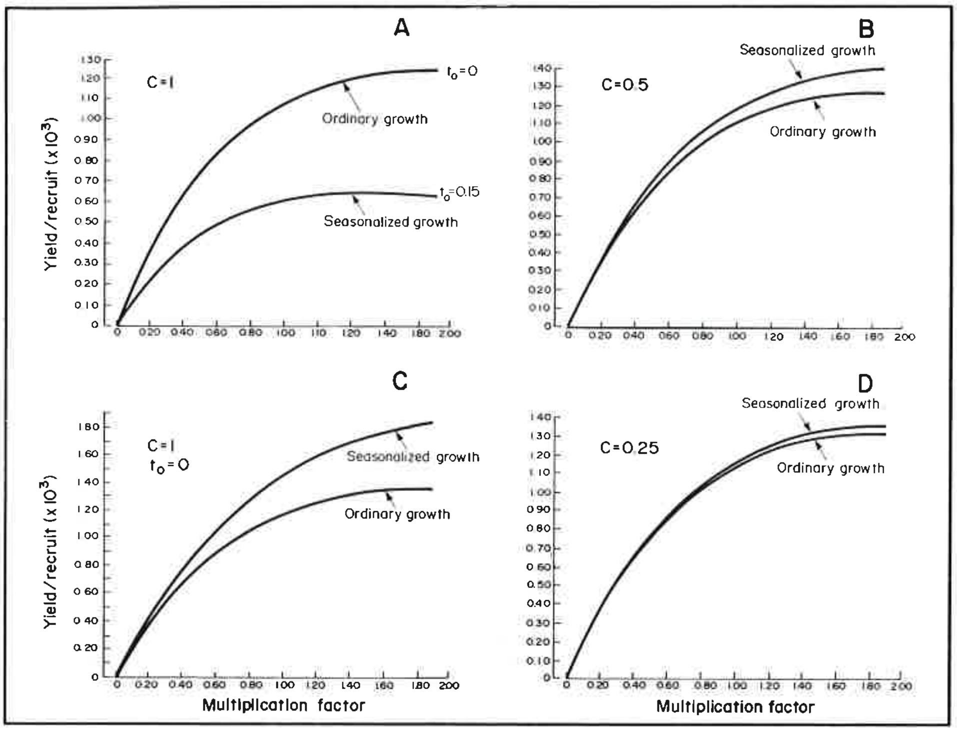

Fig. 2. Comparison of seasonalized and nonseasonalized yield per 10,000 recruit curves for a short-lived fast-growing species. Example A of Table 5.
A: C = 1.00, definition 1 of t_o (t_o = 0.151)
B: C = 1.00, definition 2 of t_o (t_o = 0.0)
C: C = 0.50, definition 2 of t_o (t_o = 0.0)
D: C = 0.25, definition 2 of t_o (t_o = 0.0)

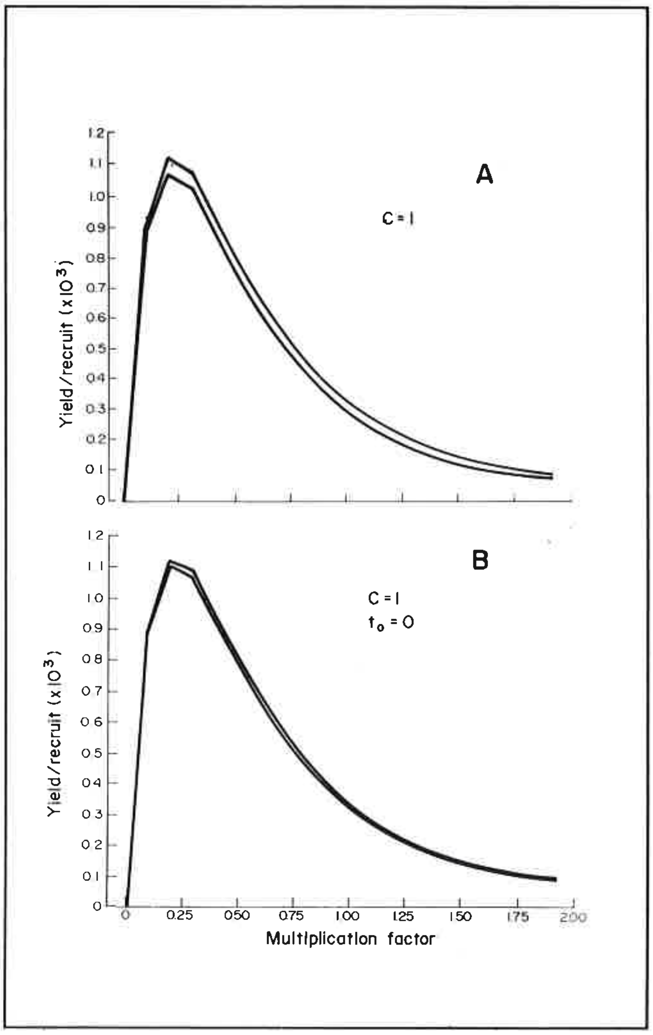

Fig. 3. Comparison of seasonalized and nonseasonalized yield per 10,000 recruit curves for a long-lived slow-growing species.
A: C = 1.00, definition 1 of t_o (t_o = 0.151)
B: C = 1.00, definition 2 of t_o (t_o = 0.0)

Table 5. Parameters used in four simulations.

Example	A	B
Number of years in life span	2	10
L_∞	100	100
Curvature parameter (K)	1.5	0.1
Natural mortality (M)	3.0	0.2
Seasonality parameter (C)	1.0	1.0
Winter point (t_w)	0.8	0.8
t_o, nonseasonalized	0.	0
t_o, season (Definition 1)	0.151	0.151
t_o, season (Definition 2)	0	0

As might be seen, seasonality has little influence on the results when C is small, even in the case of fast-growing species with high mortality.

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