

Correction to the Beverton and Holt Z-Estimator for Truncated Catch Length-Frequency Distributions

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Abstract

A method for estimating the instantaneous mortality rate (Z) is presented which was developed from a truncated equation for average length. The model has zero bias at equilibrium, but has no explicit solution for Z so that its solution requires a numerical method.

Introduction

Several simple methods for estimating mortality from length-frequency data have been adapted for use in tropical fisheries (see contributions in Pauly and Morgan 1987). Because of its simple computational formula, the equation of Beverton and Holt (1956) has been widely used for calculating total instantaneous mortality rates (Z). However, the Beverton and Holt model, which assumes steady-state conditions, also assumes that species exhibit infinite exploitable life spans. In the tropics, most species tend to have high rates of growth and natural mortality and are therefore relatively short-lived. Tropical fisheries are moreover usually characterized by numerous and heterogeneous artisanal fleets which have restricted operational ranges and highly selective gears. These conditions result in catch length-frequency distributions which are usually truncated at lower (L') and upper (L_u) size class boundaries. This results in biases for estimates of Z based on the Beverton and Holt estimators because the maximum age in the catch is very small, much less than the high ages for which the assumption of "infinite" age makes no differences. This suggested the need for a model more representative of the availability and selectivity patterns observed in tropical artisanal fisheries and of typical catch compositions in general.

Truncated Average Length Equation

Because the assumption of an infinite exploitable life span does not hold in many fisheries, Ehrhardt and Ault (in press) develop and fully explore a modified mortality formulation reexpressing the Beverton and Holt (1956) model for the average length of fish in the catch as a censored distribution truncated with a $(t_u - t')$ -bounded exploitable life span. In the new model, the hypothetical infinite life span assumed by Beverton and Holt is restricted such that the upper bound of the life span is defined by an age t corresponding to a length L_u , while the lower bound is defined by an age t' corresponding to the length L' , i.e., the lowest size at which the probability of capture is equal (or at least very near) to one. Then, mean weighted length, \bar{L} , in the catch is expressed as

$$\bar{L} = \frac{\int_{t'}^{t_u} N_t L_t dt}{\int_{t'}^{t_u} N_t dt} \quad \dots 1$$

The solution of (1) is based on the premise that growth can be adequately expressed by a simple von Bertalanffy growth equation,

$$L_t = L_{\infty} (1 - e^{-K(t-t_0)}) \quad \dots 2$$

where L_t is length at age t ; L_{∞} the asymptotic length; K the growth coefficient, and; t_0 is a fitted parameter corresponding to $L_t = 0$. Another premise is that

population size can be expressed by an exponential mortality model,

$$N_t = R'e^{-(F+M)(t-t_0)} \quad \dots(3)$$

where N_t is abundance in numbers of individuals at age t ; R' the recruitment to the exploited stock; F the instantaneous fishing mortality rate; and M the instantaneous natural mortality rate. Integration of (1) results in an expression for survival during the exploitable life span ($t-t'$) obtained after some simplifications as

$$\left[\frac{L_\infty - L_\lambda}{L_\infty - L'} \right]^{\frac{Z}{K}} = \frac{2(L' - \bar{L}) + K(L_\infty - \bar{L})}{2(L_\lambda - \bar{L}) + K(L_\infty - \bar{L})} \quad \dots(4)$$

If independent estimates of the growth parameters L_∞ and K are available, and if L' , L_λ and \bar{L} can be estimated from length-frequency distributions, then (4) contains only one unknown, the instantaneous total mortality rate Z . However, (4) does not generate an explicit solution for Z , hence total mortality rate must be obtained by successive approximations, i.e., iteratively^a.

Improved estimates of L_λ can be obtained using extreme value theory (Formacion et al. 1991). The theory provides a method to estimate L_λ from the set of maximum length (L_m , where $m = 1, 2, \dots, m$) of a series of length-frequency samples.

An Application

For analysis purposes, growth parameters were selected for two species: the relatively fast-growing and short-lived Chilean hake (*Merluccius gayi*) and the slow-growing long-lived gag grouper (*Mycteroperca microlepis*) (Table 1). Relative bias under equilibrium conditions was estimated as $((Z_{estimated} - Z_{input}) / Z_{input}) * 100$. For transitional cases, the per cent increase in annual fishing mortality rate was computed as $F(t+1) = F(t) + (\Delta F * F(t))$.

Of the two models, the Beverton and Holt model was consistently positively biased, even under equilibrium conditions. The bias was less for the longer-lived grouper than

^aA program to solve equation (4) on IBM PC and compatibles can be obtained by sending a formatted diskette to the first author. Note also that equation (4) and a routine for its solution are incorporated into the FISAT package of FAO and ICLARM, described elsewhere in this issue of Fishbyte.

Table 1. Parameters of the two types populations utilized in the mortality model and transitional simulation analysis.

Parameters	Units	Hake (<i>Merluccius gayi</i>)	Grouper (<i>Mycteroperca microlepis</i>)
L_∞	cm	63.90	129.00
W_∞	kg	1.35	25.03
K	year ⁻¹	0.33	0.12
t_0	year	0.04	-1.13
M	year ⁻¹	0.38	0.20
t_r	year	2.50	2.50
t'	year	2.50	2.50
t	year	5.00	13.00
L'	cm	35.53	46.13
L_λ	cm	51.47	105.98
M/K	-	1.20	1.60

for the hake; the bias of (4) was negligible for both stock types (Fig. 1). If various rates of changes in increasing fishing mortality are applied against a grouper stock initially in equilibrium, the induced perturbations cause a propagation of bias of (4) (Fig. 2). The bias is exacerbated for the Beverton and Holt model.

Discussion

The Beverton and Holt (1956) model was developed for long-lived slow-growing fish stocks, and it has been

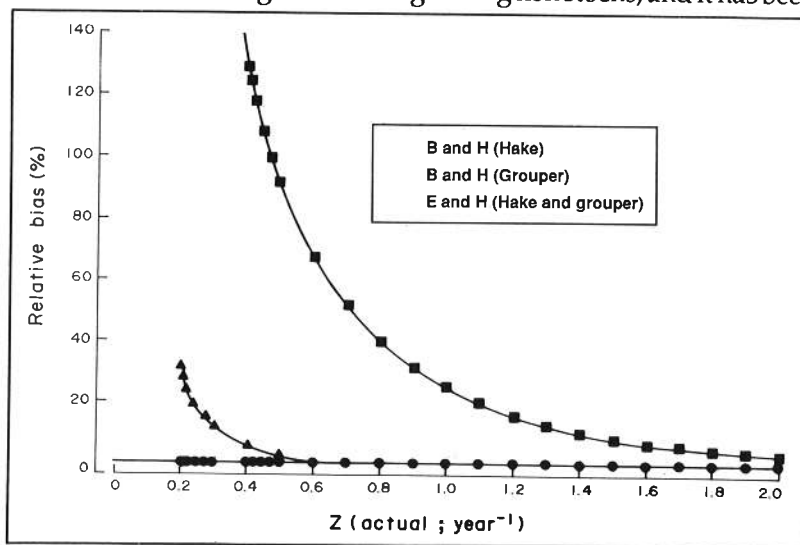


Fig. 1. Showing the bias of Beverton and Holt (B and H) Z-estimator for hake and grouper (see Table 1), compared with the lack of bias of the estimator proposed here.

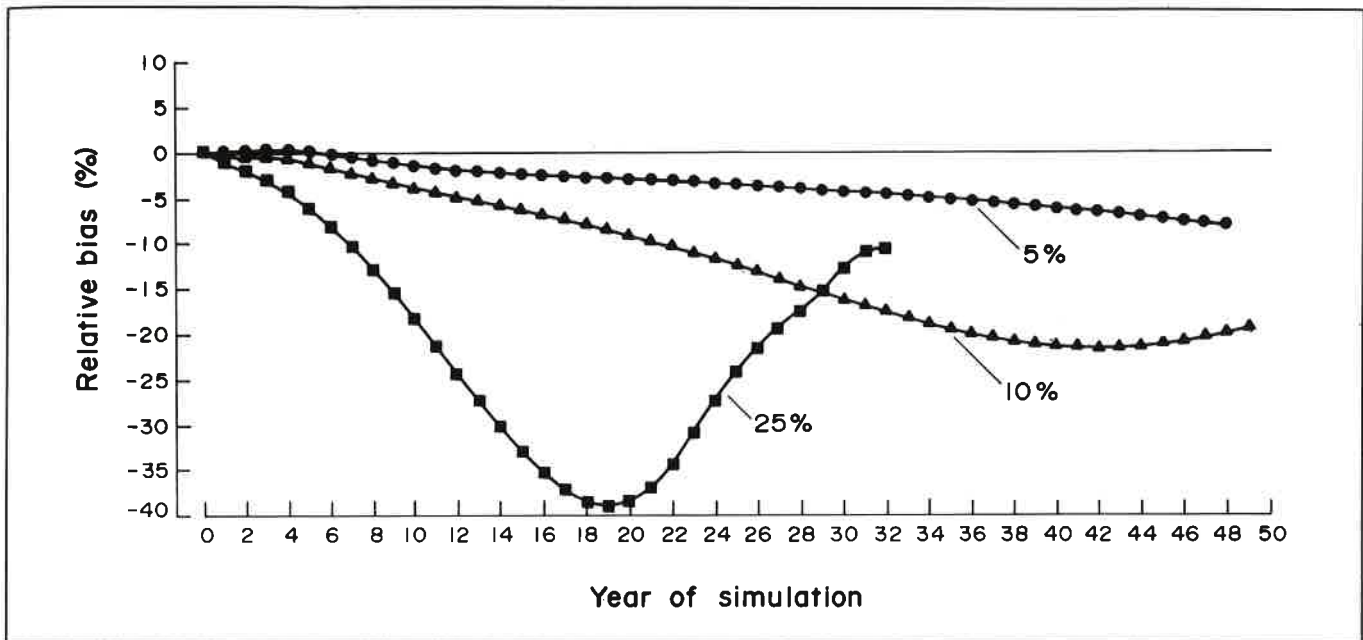


Fig. 2. Performance of the new method [equation (4)] for estimating total instantaneous mortality rate from average size of fish in the catch under three transitional rates of fishing mortality, applied to the grouper stock.

frequently used in developing fisheries because of its minimal data requirements. In many instances, the model has been applied to fast-growing species exhibiting high mortality. Consequently, the associated bias at low to intermediate Z can be significant.

In the case of species exhibiting bounded exploitable life spans, the Beverton and Holt Z -estimator always overestimated the true value of Z , primarily because the model was developed under the simplifying assumption that maximum age was infinite. Of course, bias in Z will be close to zero when exploitable life span approaches the biological life span of longer-lived species. Reduction in exploitable life span is obviously more significant when minimum fish size requirements are used for managing shorter-lived species. Therefore, estimates of Z derived from Beverton and Holt's (1956) procedure for these species will always be positively biased.

[Ault and Fox (1989), who investigated the effects of spawning and recruitment patterns in tropical and subtropical fish stocks on various assessment methods found an even stronger bias for the Z -estimator of Ssentengo and Larkin (1973)].

Bias of the Beverton and Holt Z -estimator decreased as fishing mortality rates, F , increased. This resulted from the upper limits of the population length distribution being numerically decimated at higher

mortality levels. It is expected, therefore, that in fisheries where F is controlled by quotas or limitation of fishing effort, the Z -estimator will always be positively biased.

The mortality model expressed by (4), by considering catch mean length as a function of L' and L_{∞} , is not affected by bounding exploitable life spans, and hence, under similar conditions, is more robust than the Beverton and Holt estimator.

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