basic parameters, C, the ratio of Lc (the length of first capture) to L∞ (the asymptotic length) in the von Bertalanffy growth equation, and the ratio F/H, the ratio of fishing to natural mortality. Different tables are given for different values of M/K (the ratio of natural mortality to the parameter in the growth equation describing the rate at which the fish approaches its asymptotic length). This form of presentation is particularly appropriate when some of the new length-based modifications of the traditional age-structured models (e.g., those of Beverton and Holt) are being used, since these often give results in terms of the ratios of parameters, rather than absolute values.

Table 1 gives, for sets of values of C and M/K, the values of F/H corresponding to the maximum yield per recruit (F/M)_{\text{MAX}} (shown by an asterisk in the Beverton and Holt tables) and to the point where the marginal yield per recruit, is 10% of the value at very low fishing intensities, (F/M)'_{0.1}.

The latter values are insensitive to variations in M/K, though increasing with increasing values of C, i.e., the larger the fish are, as a proportion of their maximum length, the more it pays to fish hard. The table also tends to confirm a rough rule of thumb that the optimum fishing intensity is around the point where fishing and natural mortality are equal.

References

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ESTIMATION OF NATURAL MORTALITY RATES FROM SELECTIVITY AND CATCH LENGTH-FREQUENCY DATA

by

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In conventional catch curves, the logarithms of the relative abundances of successive age groups are plotted against age, in order to obtain an estimate of the total instantaneous mortality rate, Z, from the slope of the descending right hand arm of the plot. Values to the left of the descending arm of the graph, reflecting catches of incompletely retained fishes are ignored.

Pauly (1982, 1983) has described how length-frequency distributions can be converted to age-structured catch curves in which the observed frequencies in successive length groups are divided by the time required for a fish to grow through each length group and plotted against the estimated age of the mid-
points of successive length groups from the slope of the resulting length-converted catch curve, an estimate of $Z_i$ is derived. This value is compounded of the fishing mortality rate ($F$) generated by the gear in question plus the natural mortality rate ($M$). Usually, this method is applied only to length groups which are larger than the size ($l'$) at which the fishes become fully retainable by the gear and is inapplicable to gill-nets and other gears which exhibit a bell-shaped selection curve or to length groups which are within the selection ogive of gear such as trawls.

The method now proposed is based upon the premise that for any fishing gear, the catch ($N$) of each successive size group over the entire size range of the catch is a function of the probability of retention ($P_i$) of that size group in the fishing gear and the relative abundance ($A$) of that size group in the population. Also, the total mortality coefficient ($Z_i$) observable between successive length groups in an exploited stock is the sum of the natural mortality coefficient plus the product of the probability of retention, $P_i$, and the prevailing fishing mortality coefficient ($F$).

Thus, for the $i$-th length group

$$Z_i = M + P_i F$$

Thus, a regression of estimates of $Z_i$ against $P_i$ should yield a regression of slope $F$ and $Y$-axis intercept equal to $M$.

**Methodology**

The requirements for implementing the method are as follows:

1. **Unbiased estimates of the length composition** of the catch averaged over a year.

2. **Estimates of the probability of retention** ($P_i$) at successive lengths ($L_i$), acquired independently of the estimates of the mean annual length composition. (See Hamley [1975] and Pope [1975] for reviews of various methods for estimating selection curves.)

3. **Estimates of parameters of the von Bertalanffy growth function**, $L_{oo}$, $t$ and $K$. If $t$ is unknown, it can be set at zero.

The analytical procedure is illustrated in Table 1 and Figs. 1 & 2 by a hypothetical example from a gill net fishery. The procedure is as follows:

1. **Tabulate the annual average length frequencies** ($N_i$) of the catch and the probabilities of retention ($P_i$) of successive length groups ($L_m$) (the subscript $m$ refers to the mid-point of a length group).

![Fig. 1. Length-converted catch curve for fishes exploited by a single size of gill net in which values of $\log_{e}R_m^*A_m/\Delta t_{i,i+1}$ are plotted against the relative ages $(t_m - t_0)$ attained at the mid-points of successive length groups. Note the change in slope resulting from mesh selectivity. Data from Table 1.](image)

**Fig. 2.** Regression of coefficients of mortality, $Z_i$, between successive length groups against the probabilities of retention, $P_i$, at the median points between those successive length groups. Data from Table 1.
2. Calculate the apparent relative abundances \( A \) of successive length groups within the selection curve as \( A = N / P \).

3. Calculate the ages (in years) at the start \( t_i \), mid-point \( t_{i+1/2} \) and end \( t_{i+1} \) of successive length groups and the time required \( \Delta t_i, i+1 \) for a fish to grow through each length group.

4. Divide \( A \) by \( \Delta t_i, i+1 \) to get the true relative abundance \( R_m \) of the fish in each length group (Fig. 1 shows a length-converted catch curve in which \( R_m \) is plotted against relative age \( t-t_i \). Note the sigmoid shape generated by mesh selectivity).

5. Calculate the survival rate between successive length groups, \( S = R_{m+1} / R_m \) and the annual coefficients of total mortality, \( Z_i = \log S / \Delta t_i, i+1 \) between successive mid-lengths.

6. Plot \( Z_i \) against \( P_i \) to obtain estimates of \( F \) and \( M \), using the equation \( Z_i = P_i F + M, \) and obtain an estimate of \( M \) (Fig. 2). Estimate a joint confidence region for \( M \) and \( F \) (see any statistics textbook).

Results and Discussion

The result of the hypothetical example from a gill-net fishery shows that the method will work given representative estimates of the annual average length-frequency composition of the catch and a good representation of a bell-shaped selection curve or a selection ogive. However, the method is sensitive to chance variations in catch or minor variations in catchability and will become increasingly so as \( P \) approaches zero. It is suggested that estimates of \( Z \) based on values of \( N \) amounting to less than 2% of the catch should not be used in the regression.

An important variation on this technique is possible if the estimates of \( N_i, A \), and \( R \) (as in Table 1) are derived from experimental fishing with sampling gear which has a retention range which is different from that of the commercially-utilized gear, and if both of the selection curves are known. For example, if the sampling gear had a smaller mesh size than is used commercially, the calculations in Table 1 would be the same but the estimates of \( Z_i \) in successive length groups would

Table 1. Hypothetical example \((K = 0.5, L_{50} = 30, M = 1.00, F = 2.00)\) showing steps in derivation of estimates of total mortality rate \( (Z_i) \) from catches \( (N_m) \) in successive length groups \( (L_m) \) by calculating apparent abundance \( (A_m = C_m / P_m) \) and true relative abundance \( (R_m = A_m / \Delta t_i, i+1) \). Survival rate \( S_i = R_{m+1} / R_m \) and \( Z_i = \log S_i / \Delta t_{m+1} \). See text for details. The ages, \( t_i \), of successive lengths are the relative ages which are calculated by assigning a zero value to the parameter, \( t_0 \), of the von Bertalanffy growth function. The subscripts \( i \) and \( m \) refer to the beginning and midpoint of a length group.

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be plotted against probabilities of retention, \( p_i \), of successive lengths, \( L_i \), in the commercial gear. In such a case, estimates of \( z_i \) pertaining to lengths below the retention range of the commercial gear (where \( p_i = 0 \)) are direct estimates of natural mortality \( M \).

Putting the foregoing in a different form, if estimates of \( R \), and thus \( Z_i \), can be obtained for length groups outside of (usually smaller than) the retention range of the commercially utilized fishing gear, estimates of the natural mortality rate, \( M \), of the unexploited size groups can be directly obtained from the sample length-frequency distributions.

**Conclusion**

Data which should, in theory, be available in any fisheries laboratory can be used to obtain routine estimates of \( M \) using this method. However, in practice, it has proven to be difficult to obtain suitable data for application of the model and data sets are now sought whereby the robustness and/or sensitivity of the method might be tested. If a time series of data is available, it might be possible to document changes, if any, in the value of \( M \) in response to development of the fishery and consequent changes in the composition of multi-species stocks.

I thank Dr. J.M. Hoenig for some useful comments on an earlier draft of this paper.

**References**


